Extension: 56421 Email: gychen@math.nctu.edu.tw

Prof. Guan-Yu Chen / Department of Applied Mathematics

Stochastic process, Markov chain mixing, Cutoff phenomenon

The Markov chain Monte Carlo (briefly, MCMC) method is a well-designed algorithm in sampling probability measures on discrete sets. Along with the Metropolis-Hasting algorithm, one may implement the MCMC method only with the local information of the targeted distributions, say the relative ratio, but without the information of the normalizing constant. When the MCMC method is simulated, it is important to select a (deterministic or random) time, say T, to stop the algorithm for sampling. Theoretically, the stopping time T can be the mixing time or the coupling time but none of them is easy to achieve.

The cutoff phenomenon is a phase-transit phenomenon in the evolution of Markov chains. This concept was introduced by Aldous and Diaconis in early 1980s in order to catch up the observation that the distribution of Markov chain is far from its stationarity before a time S and, after a relatively short period compared with S, the distribution turns out to be almost the limiting distribution. When a cutoff exists in a MCMC algorithm, the time S can be a good candidate for the stopping time of algorithm.

The MCMC method arises in many disciplines including the statistic physics, computer science, molecular biology, mathematical finance and more. From the viewpoint of interdisciplinary research, the underlying machinery can be very complicated, e.g. random walks on disordered random media and Markov processes on compact Riemannian manifolds, and a quick formula on the stopping time T and the cutoff time S will be very challenging but highly expected.

The following are some future perspectives we are interested in.

1) Spectral analysis of graph connection Laplacian
2) Random walks on random media
3) Inhomogeneous Markov chains

Figures: Those figures display some distance functions. From left to right, they are respectively Markov chains (1)without cutoff; (2)with cutoff; (3)with one bottleneck; (4)with two bottlenecks.