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# Prof. Wen-Wei Lin / Department of Applied Mathematics

## Numerical Analysis, Matrix Theory and Computation, Optimization, Computational Conformal Geometry

We are a research group on "Numerical Algebra and Geometry" in the ST Yau Center and Applied Math Department , NCTU. Our primary research interests include the following two parts:

**(a) Eigendecomposition and Fast Eigensolver for 3D Maxwell's equations.** We use finite difference Yee's scheme to discretize the Maxwell's equations with simple cube (SC) or face-centered cube (FCC) periodic domain into generalized or rational eigenvalue problems. We explicitly derive the eigendecomposition of the double-curl operator and SVD of the single curl operator. Then we develop a FFT-based algorithm for compact representations of $\nabla\times\nabla\times$ and $\nabla\times$ operators and propose null-space free methods with Newton-type and nonequivalence deflation technique for solving Maxwell's equations. We compute the band structures of (I) Dielectric materials, where $\varepsilon > 0, \mu > 0, \xi = \zeta = 0$; (II) Dispersive metallic photonic crystals with Drude or Drude-Lorentz models, where the permittivity $\varepsilon = \varepsilon(\omega)$ depends on the high-frequency $\omega$ with some poles; (III) Left-handed materials or negative-index material (complex media), where $\varepsilon < 0, \mu < 0, \xi, \zeta \neq 0$ are chiral or pseudo-chiral parameters. All trivial zero eigenvalues caused by the degenerate curl operator have been successfully deflated in our fast eigensolvers. FFT-based fast eigensolvers in Matlab code are developed to compute various band structures of I, II and III efficiently, reliably and robustly.

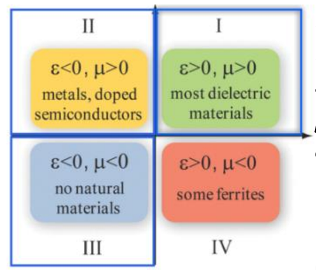
**(b) 3D Computational conformal geometry with applications**. We mainly develop efficient numerical methods to compute conformal mappings of 3D surfaces such as a genus zero closed surface, a simply connected surface with a single boundary and a closed surface of genus $\geq 1$ conformally to a sphere $S^2$, a unit disk $D_1$, a parallelogram on $R^2$ $(g = 1)$ and a polygon domain on $H^2 (g \geq 2)$. We propose a Quasi-Implicit Euler Method to a nonlinear heat equation, and the discrete Ricci flow with circle packing technique on the closed surface of $g \geq 1$ to compute the associated conformal maps ( Figure 1 and 2 ). A high-resolution 3D dynamical scanner equipped in the Lab of ST Yau Center can be used to capture various 3D human faces and the corresponding conformal disks ( Figure 3 ). We actively cooperate with industry and develop the related applications in 3D animation, retargeting/driving with texture mapping ( Figure 4 and 5 ). Furthermore, facial expression analysis, face recognition, antiquities presentation, industrial inspection, and medical image are under investigation.

*Maxwell's equations* *Frequency Domain*

$\nabla \times E = i\omega(\mu H + \zeta E),$

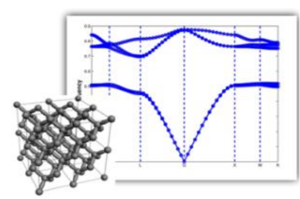
$\nabla \times H = -i\omega(\varepsilon E + \xi H),$

$\nabla \cdot (\varepsilon E) = 0, \nabla \cdot (\mu H) = 0.$

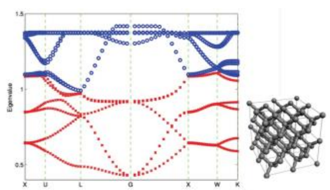



*E: electronic filed,*
*H: magnetic filed,*
*ε: permittivity,*
*μ: permeability,*
$\zeta, \xi$: magnetoelectric parameters

**I. Dielectric materials**



**II. Dispersive metallic materials**



**III. Negative index materials**

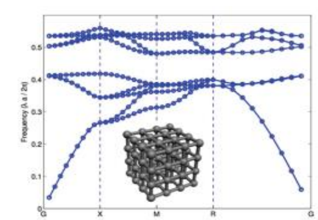


**Figure 1**

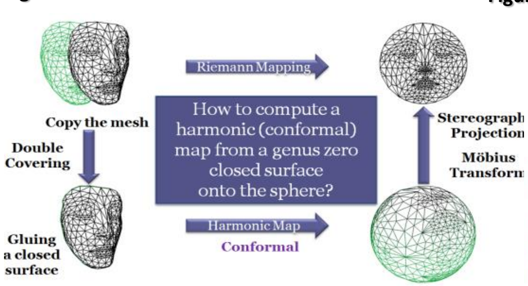



**Figure 2**

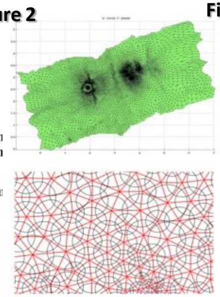


**Figure 3**

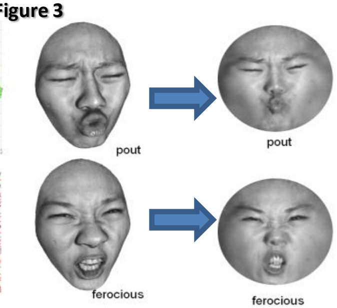



**Figure 4**

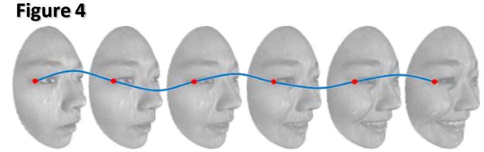
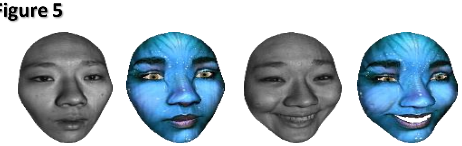


**Figure 5**



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